POLYNOMIALS

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Introduction

Video

Definition of polynomial

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial

Types of polynomials

1) Monomial :- An algebraic expression consisting of only one term is called a Monomial

Ex: 7, $7x^2$ and $3^2x^2y^4z^5$

Note: - Degree of Monomial $3^2x^2y^4z^5$ is 2+4+5= 11

2) Binomial :- An algebraic expression consisting two term is called a binomial

Ex:- 2x+3, $y^2 + 7$

3) Trinomial ;- An algebraic expression consisting three term is called a trinomial

Ex:- $2x^2$+7y+8, 2a+5b+7

Polynomial in one variable

An algebraic expression P(x) is of the form a_0+ a_1x+ a_2x^2+……+ a_nx^n, where a_0, a_1, a_2,…… a_n are all real numbers and n is any non-negative integer, is called polynomial in one variable x .

Note:- Degree of polynomial is the highest power of the variable in a polynomial

Classification of Polynomial by degree

1) Linear polynomial:- A polynomial of degree one, in one variable is called a linear polynomial

Ex:- 3x+2, $\sqrt{2}$x+5

Note :- ax+b is the general form of linear polynomial

2) Quadratic Polynomial;- A polynomial of degree two, in one variable is called a quadratic polynomial

Ex:- $2x^2$+7x-1

Note :- ax^2+bx+c is the general form of quadratic polynomial

3) Cubic Polynomial :- A polynomial of degree three, in one variable is called a cubic polynomial

Ex:- $7x^3$+$2x^2$+3x+1

Note :- a$x^3 + bx^2 + cx + d$ is the general form of cubic polynomial

Home Work

1) Which of the following expression are polynomials in one variable and which is not?

a) $4x^2$=3x+7 b) $y^2 + \sqrt{2}$ c) $3\sqrt{t} + t\sqrt{2}$

d) y+$\frac{2}{y}$ e) $x^{10} + y^3 + t^{10}$

2) Write the degree of each of the following polynomial

a) $5x^3 + 4x^2 + 7x$ b) 4-y^2 c) 5t-$\sqrt{7}$ d) 3

3) Classify the following as linear, Quadratic and cubic polynomial

a) $x^2 + x$ b) x- x^3 c) y+$y^2 + 4$ d) 1+x

e) 3t f) t^3 g) $7x^3$

Zeroes of a polynomial

The value for which the any polynomial will be zero than that value is called the Zero of the polynomial

Video

Find the zero of the polynomial in each of the following cases

1) $p(x) = x+5$ 2) $p(x) = x-5$

3) $p(x) = 2x+5$ 4) $p(x) = 3x-2$

5) $p(x) = 3x$ 6) $p(x) = ax$

7) $p(x) = cx+d$

Find the value of the polynomial $5x-4x^2+3$ at

1) x=0 2) x=-1 3) x=2

Sol:- $p(x) = 5x-4x^2+3$

Put x=-1

$p(-1) = 5(-1) -4(-1)^2+3$

$= -5 -4+3$

$= -9+3$

$= -6$

Find p(0), p(1) and p(2) for each of the following polynomials

1) p(y) = $y^2 - y + 1$

2) p(t) = 2+t+2$t^2 - t^3$

3) p(x) = x^3

4) p(x) = (x-1)(x+1)

Verify whether the following are zeroes of the polynomial indicated against them

1) p(x) = 3x+1 , x=$\frac{1}{3}$

2) p(x) = 5x-π , x= $\frac{4}{5}$

3) p(x) = $x^2 - 1$, x = 1,-1

(4) p(x) = (x+1)(x-2), x=-1,2

5) p(x) = x^2, x=0

6) p(x) = lx+m , x = $\frac{-m}{l}$

7) p(x) = 3x^2-1, x = - $\frac{1}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$

8) p(x) = 2x+1 , x= $\frac{1}{2}$

REMAINDER THEOREM

Let P(x) be any polynomial of degree greater than or equal to one and let ‘a’ be any real number. If p(x) is divided by the linear polynomial x-a, then the remainder is p(a).

Proof :- Let p(x) be any polynomial with degree greater than or equal to one. Suppose that when p(x) is divided by x-a, the quotient is q(x) and the remainder is r(x).

i.e $p(x) = (x-a).q(x) + r(x)$ D = dXq+r

put x=a

$p(a) = (a-a).q(a) + r(a)$

$P(a) = 0.q(a) + r(a)$

$p(a) = r(a)$

1) Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

a) x+1 b) x - $\frac{1}{2}$ c) x d) x + π e) 5+2x

Sol. P(x) = $x^3 + 3x^2 + 3x + 1$

5+2x = 0

2x =-5

x = $\frac{-5}{2}$

p($\frac{-5}{2}$) = $\left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3(\frac{-5}{2})+1$

= $\frac{-125}{8} + 3\left(\frac{25}{4}\right) - \frac{15}{2} + 1$

= $\frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

= $\frac{-125}{8} + \frac{75}{4} x\frac{2}{2} - \frac{15}{2}x\frac{4}{4} + 1x\frac{8}{8}$

= $\frac{-125+150-60+8}{8}$

= $\frac{-185+158}{8}$

= $\frac{-27}{8}$

∴ 5+2x is not a factor of p(x)

2) Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by x – a

3) Check weather 7+3x is a factor $3x^3 + 7x$

4) Determine which of the following polynomials has (x + 1) is factor

a) $x^3 + x^2 + 1$ b) $x^4 + x^3 + x^2 + x + 1$

c) $x^4 + 3x^3 + 3x^2 + x + 1$ d) $x^4 - x^3 - \left(2 - \sqrt{2}\right)x + \sqrt{2}$

Sol:- p(x) = $x^4 - x^3 - \left(2 - \sqrt{2}\right)x + \sqrt{2}$

x +1 =0

x = -1

p(-1) = $(-1)^4 - (-1)^3 - (2 - \sqrt{2})(-1) + \sqrt{2}$

= 1 + 1 + 2 - $\sqrt{2}$ + $\sqrt{2}$

= 4

∴ x + 1 is not a factor of p(x)

Use the factor theorem to determine whether g(x) is a factor of p(x) in each of the following case

1) p(x) = $2x^3 + x^2 - 2x - 1$, g(x) = x+1
2) P(x) = $x^3 + 3x^2 + 3x + 1$, g(x) = x+2
3) P(x) = $x^3 - 4x^2 + x + 6$, g(x) = x-3

Find the value of k, if x-1 is a factor of p(x) in each of the following cases

1) p(x)= $x^2 + x + k$
2) p(x) = $2x^2$+kx+$\sqrt{2}$
3) p(x) = k$x^2 - \sqrt{2}x + 1$
4) p(x) = k$x^2 - 3x + k$

Sol:-) p(x) = k$x^2 - \sqrt{2}x + 1$

x-1=0

x=1

p(1) = k$(1)^2 - \sqrt{2}(1) + 1$=0

k(1) -$\sqrt{2}$ +1 =0

K -$\sqrt{2}$ +1 =0

K = $\sqrt{2} - 1$

Factorize

a) $12x^2 - 7x + 1$ b) $2x^2 + 7x + 3$

c) $6x^2 + 5x - 6$ d) $3x^2 - x - 4$

c) $6x^2 + 5x - 6$

Sol:- $6x^2 + (9x - 4x) - 6$

$6x^2 + 9x - 4x - 6$

3x(2x+3) -2(2x+3)

(2x+3)(3x-2)

6X6=36

36= 12x3

36 = 6x6

36 = 9x4

d) $3x^2 - x - 4$

$3x^2$ - (4x-3x)-4

$3x^2$ -4x +3x -4

x(3x-4) +1(3x-4)

(3x-4)(x+1)

3 X4=12

12= 2X6

12= 4X3

Find the remainder obtained on dividing $x^3 + 1\ by\ x + 1$

Sol:- x+1) $x^3 + 0x^2 + 0x + 1(x^2$-x+1

$x^3 + x^2$

\- -

$-x^2 + 0x + 1$

$-x^2$ - x

\+ +

---

x + 1

x + 1

\- -

0

$\frac{x^3}{x} = x^2$

$x^2(x+1) = x^3 + x^2$

$\frac{-x^2}{x} = -x$

-x(x+1)= $-x^2 - x$

$\frac{x}{x} = 1$

1(x+1)= x+1

Factorize $x^3 - 23x^2 + 142x - 120$

Sol:- first find the factor of 120 are ±1, ±2, ±3, ±4, ±5, ±6, ±8, ±10, ±12, ±15, ±20, ±24, ±30, ±60

By trial method

P(x) = $x^3 - 23x^2 + 142x - 120$

P(1) = $(1)^3 - 23(1)^2 + 142(1) - 120$

= 1 -23 +142 -120

= 143 – 143

= 0

∴p(1) = 0 then x-1 is a factor of p(x)

x-1) $x^3 - 23x^2 + 142x - 120$(x^2-22x+120

x^3 - x^2

\- +

\- $22x^2 + 142x - 120$

\- $22x^2$ + 22 x

\+ -

120x -120

120x -120

\- +

0

$\frac{x^3}{x} = x^2$

$x^2(x-1) = x^3 - x^2$

$\frac{-22x^2}{x}$= -22x

-22x(x-1) = $-22x^2 + 22x$

$\frac{120x}{x}$= 120

120(x-1) = 120x -120

Now Find the factor of x^2-22x+120

x^2- (12x+10x)+120

x^2-12x-10x+120

x(x-12) – 10(x-12)

(x-12)(x-10)

120= 2x60

120=3x40

120= 4x30

120=5x24

120= 6x20

120= 8x15

120= 10x12

∴ the factor of $x^3 - 23x^2 + 142x - 120$ are

(x-1)(x-12)(x-10)

Home work

Factorize

a) $x^3 - 2x^2 - x + 2$

b) $x^3 - 3x^2 - 9x - 5$

c) $x^3 + 13x^2 + 32x + 20$

d) $2y^3 + y^2 - 2y - 1$

Algebraic Identities

1) $(a+b)^2 = a^2 + 2ab + b^2$ or $a^2 + b^2 + 2ab$

2) $(a-b)^2 = a^2 - 2ab + b^2$ or $a^2 + b^2 - 2ab$

3) $(a+b)(a-b) = a^2 - b^2$

4) $(x+a)(x+b) = x^2 + (a+b)x$+ab

5) $(a+b+c)^2$= $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

6) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ or $a^3 + 3ab(a+b) + b^3$

7) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ or $a^3 - 3ab(a-b) - b^3$

8) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

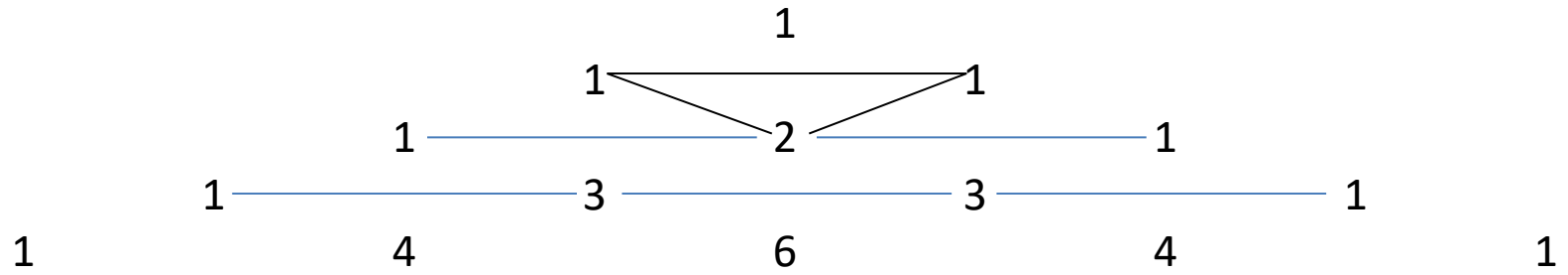
9) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

10)(a+b+c)$(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

11)If a+b+c=0 than $a^3 + b^3 + c^3$= 3abc

1) $(a+b)^0 = 1$
2) $(a+b)^1 = a+b$
3) $(a+b)^2 = a^2+2ab+b^2$
4) $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
$= a^3+3ab(a+b)+b^3$

5) $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$



a+b+c=0
a+b = -c
c.o.b.s
$(a+b)^3 = (-c)^3$
$a^3+3ab(a+b)+b^3 = -c^3$
$a^3+3ab(-c)+b^3 = -c^3$
$a^3-3abc+b^3 = -c^3$
$a^3+b^3+c^3 = 3abc$

Find the following products using appropriate identities

1) (x+3)(x+3)

Sol:- $(x+3)^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$x^2 + 2(x)(3) + 3^2$

$x^2 + 6x + 9$

2) (x-3)(x+5)

(x+a)(x+b)= $x^2 + (a+b)x + ab$

x=x, a=-3 and b=5

$x^2 + (-3+5)x + (-3)(5)$

$x^2 + 2x - 15$

3) (x-3)(x+3)

(a-b)(a+b)=$a^2 - b^2$

a=x, b=3

$x^2 - 3^2$

$x^2 - 9$

Home work

1) (x+4)(x+10)

2) (x+8)(x-10)

3) (3x+4)(3x-5)

4) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

5) (3-2x)(3+2x)

Evaluate the following products by using suitable identity

1) 102 X 106
2) 105 X 103
3) 95 X 96
4) $(98)^2$
5) 195 X 195 – 105 X 105
6) 104X96
7) 103X107

Sol;- 1) 102 X 106

(100+2)X(100+6)

(x+a)(x+b)= $x^2 + (a + b)x + ab$

x = 100, a= 2 and b= 6

$100^2 + (2 + 6)100 + 2 * 6$

10000 + 8(100) +12

10000 + 800 + 12

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Factories the following using appropriate identities

a) $25x^2 - 9y^2$

b) $25x^2 + 60xy + 36y^2$

c) $a^2 - 1 - 2b - b^2$

d) $x^2 - \frac{y^2}{100}$

e) $\frac{49a^2}{4} - 7a + 1$

f) $9x^2 + 6xy + y^2$

g) $4y^2 - 4y + 1$

a) $25x^2 - 9y^2$

$(5x)^2 - (3y)^2$

$a^2 - b^2 = (a + b)(a - b)$

a = 5x and b = 3y

(5x+3y)(5X-3Y)

b) $25x^2 + 60xy + 36y^2$

$(5x)^2$ + 2(5x)(6y)+ $(6y)^2$

$a^2 + 2ab + b^2 = (a + b)^2$

a = 5x and b = 6y

$(5x + 6y)^2$

(5x+6y)(5x+6y)

c) $a^2 - 1 - 2b - b^2$

$a^2 - (1^2 + 2(1)(b) + b^2)$

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - (1 + b)^2$

$a^2 - b^2 = (a + b)(a - b)$

a = a and b = 1+b

(a+1+b)(a-(1+b))

(a+1+b)(a-1-b)

Expand each of the following using suitable identities

a) $(x+2y+4z)^2$ b) $(2x-y+z)^2$ c) $(-2x+3y+2z)^2$

d) $(3a-7b-c)^2$ e) $(-2x+5y-3z)^2$ f) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Sol:- $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

a = $\frac{1}{4}a,\ b = \frac{-1}{2}b\ and\ c = 1$

$= \left(\frac{1}{4}a\right)^2+\left(\frac{-1}{2}b\right)^2+1^2+2\left(\frac{1}{4}a\right)\left(\frac{-1}{2}b\right)+2\left(\frac{-1}{2}b\right)(1)+2(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

Factorize

1) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
2) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$
3) $16x^2 + y^2 + 9z^2 + 8xy + 6yz + 24xz$

Sol:- $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$\left(\sqrt{2}x\right)^2 + y^2 + 4X2z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$\left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

a = $-\sqrt{2}x$, b = y and c = $2\sqrt{2}z$

$\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

(-$\sqrt{2}x + y + 2\sqrt{2}z$) (-$\sqrt{2}x + y + 2\sqrt{2}z$)

Write the following cube in expanded form

1) $(3a+2b)^3$ 2) $(a-3b)^3$ 3) $(5a-3b)^3$ 4) $(2x+1)^3$

5) $(2a-3b)^3$ 6) $\left(\frac{3}{2}x+1\right)^3$ 7) $\left(x-\frac{2}{3}y\right)^3$

Sol:- $\left(x-\frac{2}{3}y\right)^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

a = x and b = $\frac{2}{3}y$

$x^3 - 3x^2(\frac{2}{3}y)$+3x$\left(\frac{2}{3}y\right)^2 - \left(\frac{2}{3}y\right)^3$

$x^3 - 2x^2y$+3x$(\frac{4}{9}y^2)$-$\frac{8}{27}y^3$

$x^3 - 2x^2y$+$\frac{4}{3}xy^2 - \frac{8}{27}y^3$

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Evaluate the following by using suitable identities (Date:-23/04/20)

1) $(98)^3$ 2) $(99)^3$ 3) $(101)^3$ 4) $(102)^3$
5) $(999)^3$ 6) $(1001)^3$ 7) $(998)^3$

Factorize each of the following

1) $8a^3 + b^3 + 12a^2b + 6ab^2$
2) $8a^3 - b^3 - 12a^2b + 6ab^2$
3) $27\text{-}125a^3 - 135a + 225a^2$
4) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
5) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$

$a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$

a = 3p and b = $\frac{1}{6}$

$$\left(3p - \frac{1}{6}\right)^3$$

$$\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

Factorize each of the following

1) $27y^3 + 125z^3$
2) $64m^3 - 343n^3$
3) $x^3 + 64$
4) $8x^3 - 343y^3$
5) $a^3 - b^3 - a + b$
6) $(2x + 3y)^3 - (2x - 3y)^3$
7) $1 - 64a^3$
8) $8x^3 - \frac{1}{27y^3}$

$a^3 - b^3 - a + b$

$a^3 - b^3 - (a - b)$

$[(a - b)(a^2 + ab + b^2)] - (a - b)$

$(a-b)(a^2 + ab + b^2 - 1)$

$(2x + 3y)^3 - (2x - 3y)^3$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

a = 2x+3y and b = 2x-3y

$[2x + 3y - (2x - 3y)][(2x + 3y)^2 + (2x + 3y)(2x - 3y) + (2x - 3y)^2]$

$(2x+3y-2x+3y)[4x^2 + 12xy + 9y^2 + (4x^2 - 9y^2) + 4x^2 - 12xy + 9y^2]$

$(6y)[8x^2 + 18y^2 + 4x^2 - 9y^2]$

$(6y)(12x^2 + 9y^2)$

$$(2x+3y)^3-(2x-3y)^3$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$$

a = 2x and b = 3y

$(2x)^3+3(2x)^2(3y)+3(2x)(3y)^2+(3y)^3$ - $[(2x)^3-3(2x)^2(3y)+3(2x)(3y)^2-(3y)^3]$

$8x^3+36x^2y+54xy^2+27y^3-8x^3+36x^2y-54xy^2+27y^3$

$72x^2y+54y^3$

$18y(4x^2+3y^2)$

Factorize $27x^3 + y^3 + z^3 - 9xyz$

Sol:- $(3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

a = 3x, b = y and c = z

$(3x + y + z)\big((3x)^2 + y^2 + z^2 - (3x)(y) - y.z - z(3x)\big)$

(3x+y+z)$(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol;- RHS $\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$\frac{1}{2}(x + y + z)[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx]$

$\frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$\frac{1}{2}(x + y + z)2[x^2 + y^2 + z^2 - xy - yz - zx]$

$(x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$

$x^3 + y^3 + z^3 - 3xyz\ (LHS)$

LHS = RHS

Verify

1) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

2) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol:- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

RHS $(x + y)(x^2 - xy + y^2)$

x$(x^2 - xy + y^2)$+y$(x^2 - xy + y^2)$

$x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$x^3 + y^3$ (LHS)

LHS = RHS

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If x+y+z=0 than show that $x^3 + y^3 + z^3 = 3xyz$

Sol:- x+y+z=0

x+y = -z

C.O.B.S

$(x + y)^3 = (-z)^3$

$(a + b)^3 = a^3 + 3ab(a + b) + b^3$

a = x and b = y

$x^3 + 3xy(x + y) + y^3 = -z^3$

$x^3 + 3xy(-z) + y^3 = -z^3$

$x^3 - 3xyz + y^3 = -z^3$

$x^3 + y^3 + z^3 = 3xyz$

Without actually calculating the cubes, find the value of each of the following

1) $(-12)^3 + 7^3 + 5^3$

2) $28^3 + (-15)^3 + (-13)^3$

Sol:- $(-12)^3 + 7^3 + 5^3$

a = -12, b = 7 and c = 5

a + b + c = -12+7+5 = 0

a+b+c=0 than $a^3 + b^3 + c^3 = 3abc$

-12+7+5=0 than $(-12)^3 + 7^3 + 5^3$=3(-12)(7)(5)

= -1260

Home Work

1) Give the possible expression for the length and breadth of each of the following rectangles, in which their area are given

a) $25a^2 - 35a + 12$ b) $35y^2 + 13y - 12$

2) What are the possible expression for the dimensions of the cuboids whose volumes are given below?

a) $3x^3 - 12x$

3x($x^2 - 2^2$)

3x(x+2)(x-2)

b) $12ky^2 + 8ky - 20k$

If $x + \frac{1}{x} = 6$, find the value of

1) $x^2 + \frac{1}{x^2}$

2) $x^4 + \frac{1}{x^4}$

Sol:- $x + \frac{1}{x} = 6$

S.O.B.S

$\left(x + \frac{1}{x}\right)^2 = 6^2$

$(a+b)^2 = a^2 + 2ab + b^2$

a = x and b = $\frac{1}{x}$

$x^2 + 2(x)\left(\frac{1}{x}\right) + \frac{1}{x^2} = 36$

$x^2 + 2 + \frac{1}{x^2} = 36$

$x^2 + \frac{1}{x^2} = 36 - 2$

$x^2 + \frac{1}{x^2} = 34$

$x^2 + \frac{1}{x^2} = 34$

S.O.B.S

$\left(x^2 + \frac{1}{x^2}\right)^2 = 34^2$

$(a+b)^2 = a^2 + 2ab + b^2$

a = x^2 and $b = \frac{1}{x^2}$

$(x^2)^2 + 2(x^2)\left(\frac{1}{x^2}\right) + \frac{1}{(x^2)^2} = 1156$

$x^4 + 2 + \frac{1}{x^4} = 1156$

$x^4 + \frac{1}{x^4} = 1156 - 2$

$x^4 + \frac{1}{x^4} = 1154$

If 3x+2y=12 and xy=6 find the value of $9x^2 + 4y^2$

Sol:- 3x+2y=12

S.O.B.S

$(3x + 2y)^2 = 12^2$

$(a + b)^2 = a^2 + 2ab + b^2$

a = 3x and b = 2y

$(3x)^2 + 2(3x)(2y) + (2y)^2 = 144$

$9x^2 + 12xy + 4y^2 = 144$

$9x^2 + 12(6) + 4y^2 = 144$

$9x^2 + 72 + 4y^2 = 144$

$9x^2 + 4y^2 = 144 - 72$

$9x^2 + 4y^2 = 72$

Home work

1) If $x+\frac{1}{x} = 11$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$
2) If $x - \frac{1}{x} = -1$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$
3) If $x+\frac{1}{x} = \sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$
4) If 2x+3y=8 and xy=2, find the value of $4x^2 + 9y^2$
5) If 3x-7y=10 and xy=-1, find the value of $9x^2 + 49y^2$

If $a^2 + b^2 + c^2 = 20\ and$

$a + b + c = 0, find\ the\ value\ of\ ab + bc + ca$

Sol:- a+b+c=0

S.O.B.S

$(a + b + c)^2 = 0^2$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$

20+2(ab+bc+ca) = 0

2(ab+bc+ca) = -20

ab+bc+ca = $\frac{-20}{2}$

ab+bc+ca = -10

If a+b+c=9 and ab+bc+ca=40, find $a^2 + b^2 + c^2$

Sol:- a+b+c = 9

S.O.B.S

$$(a+b+c)^2 = 9^2$$

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 81$$

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$$

$$a^2 + b^2 + c^2 + 2(40) = 81$$

$$a^2 + b^2 + c^2 + 80 = 81$$

$$a^2 + b^2 + c^2 = 81 - 80$$

$$a^2 + b^2 + c^2 = 1$$

Home work

1) If $a^2 + b^2 + c^2 = 250$ and ab+bc+ca=3, find a+b+c

2) If a+b+c= 0 and $a^2 + b^2 + c^2$=16, find the value of ab+bc+ca

3) If $a^2 + b^2 + c^2$=16 and ab+bc+ca= 10 , find the value of a+b+c

4) If a+b+c= 9 and ab+bc+ca=23, find the value of $a^2 + b^2 + c^2$

If $x+\frac{1}{x} = 7$, $find\ the\ value\ of\ x^3 + \frac{1}{x^3}$

Sol:- $x+\frac{1}{x} = 7$

C.O.B.S

$\left(x + \frac{1}{x}\right)^3 = 7^3$

$(a+b)^3 = a^3 + 3ab(a+b) + b^3$

a = x and b= $\frac{1}{x}$

$x^3 + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) + \frac{1}{x^3} = 343$

$x^3 + 3(7) + \frac{1}{x^3} = 343$

$x^3 + \frac{1}{x^3} = 343 - 21$

$x^3 + \frac{1}{x^3} = 322$

Home work

1) If x- $\frac{1}{x}$=3, find the value of $x^3 - \frac{1}{x^3}$
2) If x+y=12 and xy=27, find the value of $x^3 + y^3$
3) If x – y = 4 and xy=21, find the value of $x^3 - y^3$
4) If 2x+3y=13 and xy=6, find the value of $8x^3 + 27y^3$
5) 3x-2y=11 and xy=12, find the value of $27x^3 - 8y^3$

Find the value of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ has (x-1) and (x+3) as factors.

Sol:- p(x) = $x^3 - ax^2 - 13x + b$

x-1 =0

x = 1

P(1) = $1^3 - a(1^2) - 13(1) + b = 0$

1 – a -13 + b = 0

- a + b – 12 =0

- a + b = 12 ----------(1)

P(x) =$x^3 - ax^2 - 13x + b$

x+3 =0

x = -3

P(-3) = $(-3)^3 - a(-3)^2 - 13(-3) + b = 0$

-27 – 9a + 39 +b =0

-9a + b + 12 =0

-9a + b = -12 --------(2)

- a + b = 12

-9a+ b = -12

\+ - +

8a = 24

8a = 24

a = 3

\- -a + b = 12

- 3 + b = 12

b = 12 + 3